TWO-DIMENSIONAL DISORDERED ELECTRON SYSTEMS: A NETWORK MODEL APPROACH

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We demonstrate that network models for wave mechanical systems with quenched disorder cover the physics of mesoscopic electrons. The models are constructed as a network of random scattering matrices connecting incoming to outgoing wave amplitudes. The corresponding wave dynamics is given by a discrete unitary time evolution operator. We report on three different universality classes: two-dimensional, spinless, non-chiral electrons with (O2NC) and without time reversal symmetry (U2NC), and two-dimensional, non-chiral electrons with time reversal symmetric spin-scattering (S2NC). We determine the phase diagram in the parameter space of scattering strengths. The O/U2NC models show strong localization. We find symmetry factors in localization lengths as well as multifractal exponents in agreement with theoretical predictions. The S2NC model displays a localization-delocalization transition. We determine the critical exponent of the localization length and the multifractal scaling exponent of the order parameter to be $\nu \approx 2.4$ and $\alpha_0 \approx 2.18$, respectively.

In phase coherent disordered electron systems (mesoscopic systems) interference phenomena can lead to strong fluctuations in physical quantities and localization of wave functions. Localization-delocalization (LD) transitions tuned by system parameters can be described in terms of the scaling theory [1] for broad distribution functions of physical quantities [2]. In the present work we start from a modeling of mesoscopic systems by networks of unitary scattering matrices [3] (see Fig 1). These models allow for a direct evaluation of physical quantities and of (quasi-energy) eigenvalues and eigenstates. Network models (NWMs) are paradigmatic for coherent waves in disordered media and do not refer to any particular dispersion relation. They are designed to cover essential symmetries and characteristic length scales, but are otherwise unspecific. For example, the wavelength can be identified with the lattice spacing which is, together with the wave velocity, set to unity. The elastic mean free path l_e is determined by the average reflection properties of single scattering units. NWMs can rather easily be designed for special purposes. For example, the situation of strong magnetic fields in 2D can be modeled [4,5] by suppression of forward and backward scattering in each scattering unit of Fig. 1.

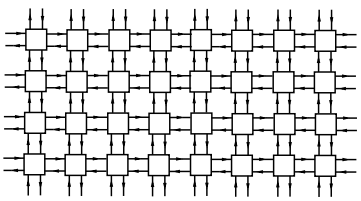

Figure 1: A two-dimensional network of scatterers (squares) connecting incoming and outgoing propagating wave amplitudes (bonds).

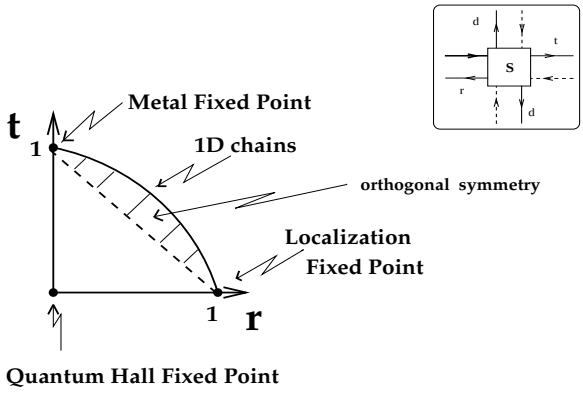

Figure 2: The parameter space of the U2NC network model: $r^2 + t^2 \leq 1$ where r and t denote transmission strengths of individual scatterers as shown in the inset. Vanishing deflection leads to decoupled 1D chains, maximum transmission (reflection) corresponds to the metal (localization) fixed point, and maximum deflection corresponds to the quantum Hall fixed point. Systems with time reversal symmetry (O2NC) have a restricted parameter space, $r + t \geq 1$, denoted as 'orthogonal symmetry'.

Consider a regular network of $\mathcal{N}$ sites and N bonds as shown in Fig. 1. The bonds carry propagating waves (incoming and outgoing) represented by complex amplitudes. On the sites unitary S-matrices map incoming to outgoing amplitudes. The elements of each S-matrix are (in general) random quantities taken from certain distribution functions, characterized by a few parameters, e.g. scattering strengths as shown in the inset of Fig. 2. Random phases (respecting symmetries) are attached to the amplitudes on the links. They simulate random distances between scatterers in realistic systems. The construction of a NWM is fixed by the choice of a type of S-matrix and a connectivity matrix C that describes how sites are connected to each other. S and C define a unitary time evolution operator U [5,6] that maps all incoming to outgoing bond amplitudes in one unit of time,

$$U\psi(t) = \psi(t+1)\,. \tag{1}$$

Here the state ψ is the vector of the N bond amplitudes. The eigenphases of U are appropriate objects for investigating local energy level statistics. The NWM depicted in Fig. 1 (with the scattering unit shown in the inset of Fig. 2) is designed to describe 2D disordered, non-interacting, spinless electrons in the absence of chiral fields. Its parameters are the transmission (reflection) strength t (r), and a deflection strength d (equal for left and right) obeying the constraint of unitarity, $t^2+r^2+2d^2 = 1$. The parameter space is shown in Fig. 2. The model is denoted as U2NC model. If the scattering is time reversal symmetric the parameter space is further restricted by $r + t \geq 1$ and the corresponding NWM is referred to as O2NC model. Under real-space renormalization the U2NC (O2NC) model has three (two) fixed points: metal, localization, and quantum Hall fixed point [4] (metal and localization fixed point). We have calculated localization lengths $\xi(M)$ in quasi-1D strip geometries of width M, and the multifractal $f(\alpha)$ spectrum of eigenstates in a square geometry. It turns out that only the localization fixed point is attractive under renormalization

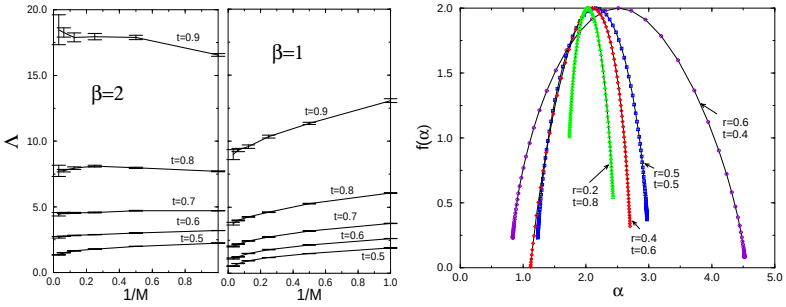

Figure 3: On the left normalized localization lengths Λ are shown as function of the inverse strip width $1/M$ for different symmetries ($\beta = 1, 2$) and different choices of the scattering strengths t (r was set to $r = 1 - t$). On the right the multifractal spectra of states in a square geometry are shown for different choices of the scattering strengths and $\beta = 2$. The scaling exponent α_0 is given as the maximum position of $f(\alpha)$.

while the others are repulsive, i.e. all states will localize for large enough system sizes. The elastic scattering length is given as $l_e = (t^2 + d^2)/(2r^2 + 2d^2)$ (for details see[8]) and by the Einstein relation a diffusive classical conductance can be defined as $g_0 = l_e/(2\pi)$. As a quantitative test we compared $\xi(M)$ with the analytic expression [7]

$$\xi(M) = l_e(\beta(2M) + 2 - \beta)\,. \tag{2}$$

Here $\beta = 1, 2$ is the usual symmetry index indicating presence (1) or absence (2) of time reversal symmetry. Equation (2) is expected to be valid in strip geometries for which $\Lambda = \xi(M)/M \gg 1$. By finite size scaling this corresponds to a 2D metallic system in the weak localization regime. In fact, we find good agreement (with an uncertainty of 10%) already for moderate values of $\Lambda \gtrsim 1$ (see Fig. 3) telling that the network model is able to describe disordered 2D electrons. As a second quantitative test we calculated the scaling exponent α_0 describing the scaling of the typical local density of states in a square geometry [9]. Such states correspond to the weak localization regime. In 2D α_0 can be related to $\Lambda(M)$ by a conformal mapping [9] as soon as Λ becomes independent of M (in practice this means $\Lambda \gtrsim 3$ for $\beta = 2$).

$$\Lambda = \frac{1}{\pi(\alpha_0 - 2)} \longrightarrow \alpha_0 - 2 = \frac{1}{2\beta g_0} \tag{3}$$

This finding coincides with the analytic result [10] obtained for large classical conductance g_0 and is consistent with our numerical results for α_0, the maximum positions in the multifractal $f(\alpha)$ spectra displayed in Fig. 3. The NWM describing 2D disordered time reversal symmetric electrons in the presence of spin scattering (S2NC model) is based on the O2NC model [11]. In addition to the scattering parameters t, r of the O2NC model a new parameter, the spin scattering parameter $s \in [0, 1]$, appears. It defines a spin scattering length $l_S(s) \in [0, \infty[$. The S2NC model shows true LD transitions in the parameter space (r, t) for non-zero values of s. The transitions can be identified by finite size scaling techniques based on the quantity $\Lambda(M)$. The phase diagram obtained for maximum spin scattering strength $s = 1$

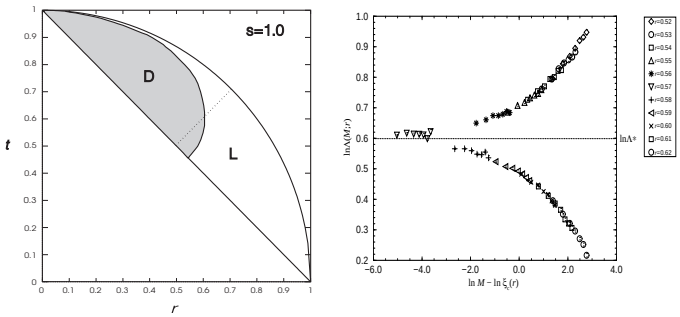

Figure 4: On the left the phase diagram of the S2NC model is shown for maximum spin-scattering strength $s = 1$. The grey area stands for the delocalized phase (D) separated from the localized phase (L). On the right a one-parameter scaling function is shown for the LD transition in the S2NC model. Data correspond to the logarithm of Λ as a function of $\ln(M/\xi_c(r))$ where $\xi_c(r)$ denotes a fitted correlation length as a function of the reflection strength r.

is shown in Fig. 4. When crossing the phase boundary in the (r, t) plane, for a fixed value of s, the quantity $\Lambda(M)$ follows one-parameter scaling [1] as can be seen in Fig. 4. Analyzing the scaling function allows for the determination of the critical exponent of the localization length, $\nu \approx 2.4$ [11]. With the help of the conformal mapping relation (see Eq. (3)) the multifractal exponent can be obtained from the fixed point value of Λ and is $\alpha_0 \approx 2.18$.

In summary, we have shown that NWMs represent universality classes of disordered wave mechanical systems. In particular, the U/O2NC models show Anderson localization. A quantitative analysis in the weak-localization regime is in reasonable agreement with known analytic results for disordered electron systems. The S2NC model describing spin-scattering exhibits localization-delocalization transitions and allows for a quantitative analysis of critical properties.

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